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Geometric Categories in Cognition

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At the scale in which we live, space is continuous. Nevertheless, our perception and cognition parse the world into categories, whether physical, like *scene* or *object*, or abstract, like *infinitesimal point* or *7*. The present study focuses on 2 categories of special angles in planar geometry, *parallels* and *perpendiculars*, and we evaluate how these categories might be reflected in adults' basic angle discrimination. In the first experiment, participants were most precise when detecting 2 parallel or perpendicular lines among other pairs of lines at different relative orientations. Detection was also enhanced for 2 connected lines whose angle approached 90°, with precision peaking at 90°. These patterns emerged despite large variations in the scales and orientations of the angle exemplars. In the second experiment, the enhanced detection of perpendiculars persisted when stimuli were rotated in depth, indicating a capacity to discriminate shapes based on perpendicularity in 3 dimensions despite large variation in angles' 2-dimensional projections. The results suggest that 2 categorical concepts which lie at the foundation of Euclidean geometry, parallelism and perpendicularity, are reflected in our discrimination of simple visual forms, and they pave the way for future studies exploring the developmental and evolutionary origins of these cognitive categories.

Public Significance Statement

In this article, we discover that 2 categories of special angles in formal geometry, *parallels* and *perpendiculars*, are robustly reflected in adults' basic angle discrimination. Our results both characterize the psychophysical properties of angle discrimination, which has been debated in prior literature, and also link vision research to the conceptual, formal, and school-relevant spatial understanding that supports abstract mathematics.

Keywords: categories, shape discrimination, spatial cognition, angle, geometry

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The full data set and analysis code are available on the Open Science Framework at: https://osf.io/wa64q/. Code for the generation of the experimental stimuli are available upon request. This work was supported by a Starting Grant of the European Research Council to Véronique Izard (FP7 Project MathConstruction 263179), by a grant from the National Science Foundation to Moira R. Dillon (DGE-1144152), and by the Norman Henry Anderson Graduate Psychology Fund to Moira R. Dillon. All authors conceived of and designed the study. Marianne Duyck programmed the stimuli. Moira R. Dillon, Marianne Duyck, and Véronique Izard collected the data. Moira R. Dillon, Marianne Duyck, and Véronique Izard performed the data analysis. Moira R. Dillon, Marianne Duyck, and Véronique Izard drafted the article, and all authors provided critical revisions. All authors approved the final version of the article. Moira R. Dillon and Marianne Duyck contributed equally and are listed in alphabetical order.

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Categories of all kinds pervade human cognition and organize our physical and mental worlds. Some categories are present early in human development and are rooted in perception. For example, very young infants show more precise discrimination (indicated by an increase in sucking frequency) of two synthetic speech sounds separated by a fixed distance in voice onset time when that distance distinguishes voiced and unvoiced stop consonants (e.g., /b/ and /p/) compared with when it does not (Eimas, Siqueland, Jusczyk, & Vigorito, 1971; Liberman, Harris, Kinney, & Lane, 1961). Moreover, even early brain responses are modulated by categories of visual stimuli: Localized regions of the visual cortex of infants as young as 4 months respond preferentially to exemplar pictures belonging to categories like *faces* and *scenes* (Deen et al., 2017). Nevertheless, such early emerging categorical processing is reshaped by experience. For example, in the first year of life, infants' auditory discrimination becomes specialized to the phonemic categories present in their native language (Kuhl, Williams, Lacerda, Stevens, & Lindblom, 1992; Werker & Tees, 1984), and children's explicit discrimination of phonemes develops reciprocally with culturally constructed reading and writing systems (see Anthony & Francis, 2005 for a review). Such complex developmental stories raise intriguing questions about the origins of our explicit categorical knowledge in adulthood: To what extent is such knowledge innate, rooted in our perceptual experiences, or acquired by explicit education or immersion in a specific language and culture? And what of those categories with no apparent perceptual origins or constraints, like the concept of *irrational number* or other such abstract concepts often found in formal mathematics? In the present study, we investigate whether two categories of special angles that lie at the foundation of formal, Euclidean geometry, *parallelism* and *perpendicularity* (Euclid, 1990/300 B.C.E.) are reflected in adults' basic angle discrimination. We do so by evaluating whether adults' discrimination acuity is enhanced around these category boundaries in a variety of perceptual contexts.

At the turn of the 20th century, Wilhelm Wundt and Hermann von Helmholtz independently suggested that angle discrimination may not be a wholly continuous process. They observed that small angles are judged to be somewhat bigger than their actual size and that big angles are judged to be somewhat smaller (von Helmholtz, 1924–25/1867; Wundt, 1897). More recent work on angle discrimination has aimed to explain and quantify this observation by suggesting, for example, that errors in discrimination reflect orientation selectivity in the visual cortex that leads to orientation distortions (Carpenter & Blakemore, 1973) or that errors are rooted in an inference about how an angle's appearance reflects its real-world size (Howe & Purves, 2005; Nundy, Lotto, Coppola, Shimpi, & Purves, 2000).

A number of studies have focused on comparing the threshold at which we are able to discriminate various angles, but the results have been mixed. In addition, such studies often incompletely controlled for a variety of visual cues in the angle stimuli, such as orientation or size, and so conclusions about angle discrimination specifically have been difficult to make (Regan & Hamstra, 1992; Snippe & Koenderink, 1994; Wenderoth & Johnson, 1984; Werkhoven & Koenderink, 1993). Chen and Levi (1996) suggested, for example, that there are different detection thresholds for angles of specific sizes, reporting more fine-grained discrimination of 90° angles in a discrimination space that otherwise follows Weber's Law. While such thresholds were measured for 12 different reference angles (ranging from 15°−180°), the angles were presented at only two different orientations, vertical or oblique, so it remains unclear whether the thresholds were generalizable to more variable orientations. Heeley and Buchanan-Smith (1996) found a similar pattern of discrimination thresholds with angles presented at random orientations, but they did not simultaneously vary the lengths of the lines that formed the angles, allowing for other, global shape cues to drive participants' performance. Regan, Gray, and Hamstra (1996), in contrast, found fairly constant thresholds of angle discrimination between 20° and 160° when angles were presented at a wide range of orientations. The authors suggest, however, that their discrepant findings might have been due to their unique methodology: Participants compared each stimulus with a reference angle size internalized through many practice trials, rather than to a physical display. The existing literature thus suggests that discrimination may be most precise around 90° angles, but it remains inconclusive.

In the present study, we measured the discrimination thresholds for many different angles using an intruder task (after Dehaene, Izard, Pica, & Spelke, 2006). We evaluated whether participants could locate an intruder that differed in angle size among five other angle exemplars of the same angle size as each other. The lines forming the angles varied considerably in their lengths and orientations to ensure that responses were made on the basis of angle alone within an individual trial. In addition, no individual lines were oriented within 10° of the horizontal or vertical to avoid any specialized angle discrimination that might occur when individual lines are orientated at those orientations (Xu, Chen, & Kuai, 2018).

We hypothesized that angle discrimination may reflect categories of parallelism and perpendicularity in two ways. First, detection thresholds may be more precise when parallels or perpendiculars serve as reference angles, compared to intruder angles of other sizes. Second, the detection of angle intruders may be asymmetric such that, for reference angles near 0°/180° and 90°, intruders whose angle size moves toward versus away from the parallel or perpendicular category boundaries will be easier to detect. This second prediction should hold when intruders cross the category boundary and also, perhaps, when intruders approach but do not cross the boundary. In Experiment 1, we find evidence for such categories. In Experiment 2, we demonstrate the robustness of these categories by imposing an additional rotation in depth in the experimental displays, dissociating three-dimensional (3D) perpendicularity from two-dimensional (2D) angle.

Experiment 1

Method

Participants. Eight adults (4 women; $M_{age} = 25$ years; range 19–28 years) participated in this experiment. The sample size was set in advance based on the maximum sample size (8 participants) used in several other studies investigating angle discrimination and presenting large numbers of trials to individual participants (Chen & Levi, 1996; Heeley & Buchanan-Smith, 1996; Regan & Hamstra, 1992; Regan et al., 1996; Snippe & Koenderink, 1994; Xu et al., 2018). In addition, stimuli were piloted in advance on three of the study's authors, and effects were robust enough to emerge in each pilot participant. As such, we also illustrate individual participant results in the figures and the online supplemental materials. All participants had normal or corrected-to-normal vision and had completed high school; most of them had also received a college or advanced degree. None were informed of the purpose and hypotheses of the study until after it was completed. This study was part of a research program approved by the Paris Descartes Ethics Committee (Conseil d'Évaluation Éthique pour les Recherches En Santé, CERES) and each participant provided informed consent prior to the experiment. Participants were paid 10€ per hour plus an additional sum, which depended on their performance (see below). On average, participants earned 77.38€ (range 75.60€–80.07€).

Design, apparatus, procedure, and stimuli. For each trial, participants were presented with six angles, five of which were identical in their angle size (hereafter referred to as the "reference" angle) and one of which differed in its angle size (hereafter referred to as the "deviant" angle). Participants were asked to identify the deviant angle. In the *connected lines* condition (Figure 1A), angles were composed of two lines that met at one end. Participants saw trials in 7 different blocks, each block with a different reference angle. The angle measures of the stimuli are summarized Table 1. The reference angles were chosen to evaluate discrimination thresholds symmetrically around 90°, and the angle differences between the reference angles and each deviant were chosen to capture potentially large differences in discrimination thresholds across the different references (if, e.g., discrimination followed Weber's Law). To probe the detection of 90° deviants specifically, ±10° were presented in the 80° and 100° reference blocks. In the *nonconnected lines* condition (Figure 1B), two lines were oriented relative to one another, but did not meet, and participants saw 8 different reference angles in separate blocks (Table 1). We again added 90° deviants in the 80° and 100° blocks. We also added 0° (parallel) deviants in the 10° block and removed from that block the −12° and −18° deviants because these values were not geometrically possible (i.e., they would have resulted in −2° and −8° deviants, which would be equivalent to 2° and 8°). Each type of deviant was presented 18 times in a random order in a block, three times at each of 6 possible target locations. The number of trials per block thus ranged from 126 to 162. Finally, despite the addition of 0° and 90° deviants in some reference blocks, those angles were not overrepresented in the experiment. For example, in the connected lines condition, a total of 756 90° exemplars were presented, compared with 810 exemplars of both 80° and 100° angles and 720 exemplars of the angles in other reference blocks (Table 1).

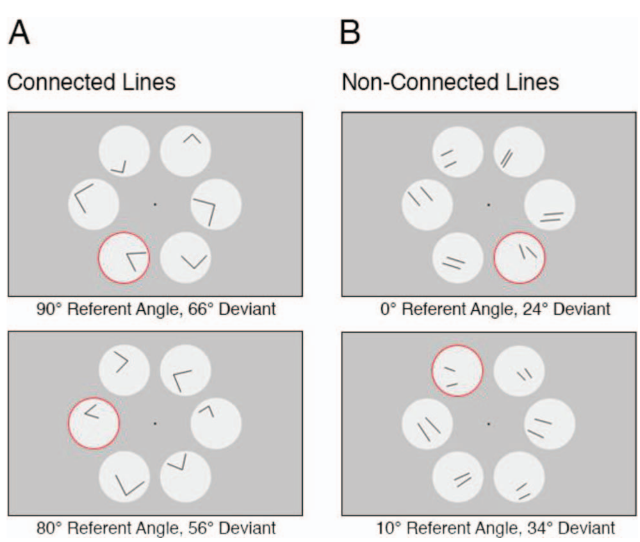

Figure 1. A. Exemplar trials from the connected lines condition. Participants showed smaller discrimination thresholds with 90° reference angles (top) than with 80° reference angles (bottom). B. Exemplar trials from the nonconnected lines condition. Participants easily distinguished nonparallel lines among parallel lines (top) but had difficulty with larger deviants when reference angles were 10° (bottom). Here, deviants differ from the reference angles by 24° (the difference used during training trials). For illustration purposes, the correct responses are circled in red. See the online article for the color version of this figure.

Participants were seated in a lit room (62 cd.m^{-2}) at eye level and 48 cm away from the center of an LCD monitor (60 Hz) subtending 44 × 32 degrees of visual angle (dva). On every trial, 6 angles appeared at the same time in one of the 6 white circular placeholders (124 cd.m^{-2}, 5 dva radius). These placeholders were equally distributed around a 11 dva radius circle centered on a central black fixation dot (0.15 cd.m^{-2}, 0.4 dva radius) on a light gray screen (92 cd.m^{-2}). While maintaining their head position with a chin rest, participants were given 5 s to look at all of the angles before the angles disappeared. Participants could respond 250 ms after the presentation onset or up to 30 s after the angles disappeared. At the beginning of each trial, the mouse cursor was positioned at the center of the screen. Participants clicked on the location of the deviant angle and received informative auditory feedback. The next trial started 500 ms after the response. After every quarter of a block, the percentage of correct responding was displayed as well as the sum earned during that block. Participants earned 0.014€ per correct response.

In both conditions, the lines that formed each angle were the same length, but these lengths varied across angles in the same display (chosen randomly from a uniform distribution between 2 and 4.4 dva). Lines were never within 10° of the vertical or horizontal axes of the screen. In the nonconnected lines condition, one of the lines was displaced relative to the other line both along it (up to its midpoint) and orthogonally (0.6–2.5 dva) to it. Finally, to ensure that all angles in each display were presented at sufficiently different orientations, each was initially assigned to either 0°, 60°, 120°, 180°, 240°, and 300° relative to the screen and then was jittered randomly between ±30°. Each angle was then translated to a random location within 0.4 dva of its placeholder's edge.

The experiment was divided into two sessions corresponding to the two conditions, which took place on different days within a two-week period. The order of conditions was counterbalanced across participants. Each session started with a short training phase consisting of two trials per reference angle displaying ±24° angle deviants. These trials were first presented with unlimited viewing time and then with the 5 s viewing time used in the actual experiment. Each block started with an introductory screen instructing participants to click on the shape that had a different angle size from the rest. This introductory screen also displayed one example of the reference angle for that block, oriented such that a vertical line would bisect the angle. The order of the blocks was random for each participant. Each session lasted approximately 3 hr, with a 15- to 30-min break after the fourth block in each condition.

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Table 1
The Angle Measures Presented in Experiments 1 and 2

Deviant angle difference	Reference angles 0	10	25	55	80	90	100	125	155
−18	18		7	37	62	72	82	107	137
−12	12		13	43	68	78	88	113	143
−10		0					90		
−7	7	3	18	48	73	83	93	118	148
−3	3	7	22	52	77	87	97	122	152
+3	3	13	28	58	83	93	103	128	158
+7	7	17	32	62	87	97	107	132	162
+10					90				
+12	12	22	37	67	92	102	112	137	167
+18	18	28	43	73	98	108	118	143	173

Note. The 25° and 155° reference angles were presented only in Experiment 1 and the 0° and 10° reference angles were presented only in the nonconnected lines condition of Experiment 1. All angle measures are in degrees.

Analyses. Participants' performance for each reference angle and for each of the connected and nonconnected line conditions was fit with down-pointing Gaussian curves constrained to chance performance (chance = 0.167) at a difference of zero degrees and perfect performance at a difference of infinity. The fitted Gaussians were used to estimate individual participants' thresholds. These thresholds corresponded to the difference in degrees between the reference and the deviant such that the participant could detect the deviant on half of the trials and was guessing on the other half (i.e., performance of 0.583, halfway between chance and perfect performance). To capture possible asymmetries in participants' responses, performance was fit separately for smaller and larger deviants. Overall thresholds for each reference were obtained by averaging these two values. Because we did not present the same number and measure of angle deviants across reference angles (e.g., we included 10° deviants in the 80° block, but not in the 55° block), such thresholds were more appropriate than raw accuracies to compare performance across reference angles. This measure was decided on after pilot testing and in advance of any data collection.

In the connected lines condition, all 8 participants performed above chance on all 7 reference angle conditions with both smaller and larger deviants (binomial tests, two-tailed, all $ps < 0.05$), yielding 112 data sets with above-chance performance. In the nonconnected lines condition, however, 17 of the 128 data sets did not significantly differ from chance, thus yielding unreliable detection thresholds (see Figure 2 for the accuracy curves of a representative participant and Figure S1 for all individual accuracy curves). To analyze the results of the nonconnected lines condition, we thus used nonparametric, rank-order tests instead of parametric tests, which replaced estimated threshold values with ranks based on thresholds' relative magnitudes. As confirmation of our results for this condition using this method, we also analyzed participants' accuracy using parametric tests, and we obtained the same results (see online supplementary materials, Table S1, Figure S2).

First, we evaluated participants' performance in the connected lines condition. Using planned one-tailed *t* tests, we examined whether participants' detection thresholds were more precise for the perpendicular (90°) reference angles compared with the 80° and 100° reference angles (the closest references to 90°, a 10° difference) and with the 55° and 125° reference angles (references farther from 90°, a 35° difference). In those same reference blocks, we then examined whether deviant detection was more precise when deviants approached or crossed the 90° boundary compared with when they did not. For example, we evaluated whether thresholds in detecting larger deviants in 80° reference blocks were more precise than detecting smaller deviants in 80° reference blocks. Since all 4 comparisons tested whether angle discrimination was influenced by the category of right angles, *p* values were adjusted for multiple comparisons using Holm's method.

For the nonconnected lines condition, we investigated whether detection thresholds were more precise for the perpendicular (90°)

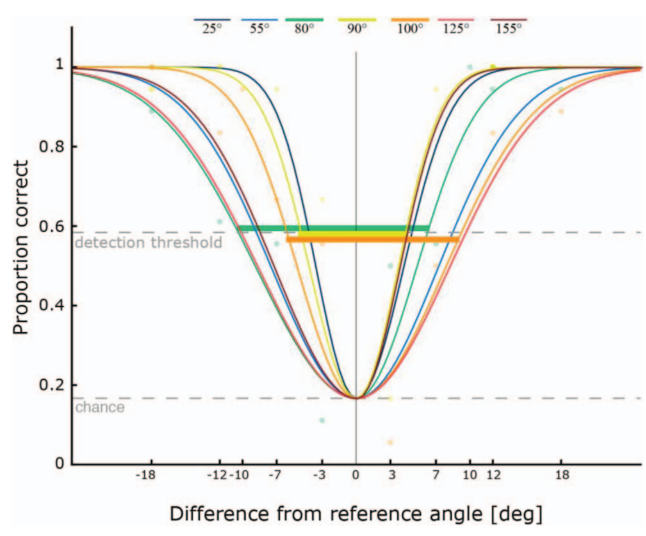

Figure 2. Model-fit curves for one participant (S2) for all reference angles in the connected lines condition of Experiment 1. Individual data points are additionally represented for the 80°, 90°, and 100° reference blocks. These curves illustrate lower thresholds in the 90° reference block (i.e., a more narrow curve, in yellow) and lower thresholds for deviants toward versus away from 90° (i.e., an asymmetry in the curves around 0° of deviation) in the 80° (in green) and 100° (in orange) reference blocks. See the online article for the color version of this figure.

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and parallel (0°) reference angles using planned Holm-corrected, one-tailed Wilcoxon signed-ranks tests. As in the connected lines condition, we compared the thresholds of detecting deviants with a 90° reference both to those with an 80°/100° reference and also to those with a 55°/125° reference. In addition, we also compared the thresholds of detecting deviants with a 0° reference to the two closest reference blocks (10° and 25°). Finally, we tested for asymmetries in detection thresholds around these categories by examining whether deviant detection was more precise when deviants approached or crossed the 90° or 0° boundaries compared to when they did not.

Results

Connected lines condition. Figure 3A displays the individual and average detection thresholds for each reference angle in the connected lines condition. All participants had similarly shaped curves, in which detection thresholds were smaller as reference angles approached 0°/180°. Strikingly, all participants also showed steep drops in their detection thresholds as reference angles approached 90°. Group-wise analyses, summarized in Table 2, corroborated these results, finding significantly more precise detection thresholds for 90° reference angles compared with 80°/100° reference angles ($t(7) = 5.47$, $p < .001$, Cohen's $d = 1.94$) and 55°/125° reference angles ($t(7) = 10.05$, $p < .001$, Cohen's $d = 3.55$). Such differences are characteristic of a categorical effect at 90°.

Moreover, we observed significant asymmetries in the blocks where the deviants crossed the 90° boundary (80° and 100° references). In these blocks, deviant detection was more precise toward 90° compared with away from it (Table 2), $t(7) = -6.29$, $p < .001$, Cohen's $d = 2.23$. However, no asymmetries were observed in the 55°/125° reference blocks, $t(7) = 0.21$, $p = .419$, Cohen's $d = 0.07$. Thus, as deviants crossed 90°, their discrimination became more precise, again signaling an influence of the perpendicular category on performance (see Figure 3B).

Nonconnected lines condition. Figure 4A displays the individual and median detection thresholds for each reference angle in the nonconnected lines condition. All participants had similarly shaped curves, in which detection thresholds were smallest at 0° compared to other reference angles (see Figure S1 for individual participant accuracy curves). Group-wise analyses corroborated these results (see Table 2), finding more precise detection thresholds for 0° reference angles compared to 10° and 25° reference angles (10°: $Z = 2.15$, $p = .016$, 8/8 participants showed the effect; 25°: $Z = 2.15$, $p = .016$, 8/8 participants showed the effect). Moreover, participants performed better with angles deviating toward versus away from 0° with both 10° and 25° reference angles (10°: $Z = 2.15$, $p = .016$, 8/8 participants showed the effect; 25°: $Z = 2.15$, $p = .016$, 8/8 participants showed the effect; see Figure 4B) suggesting that parallel lines might serve as an anchor for judgments of discrimination.

Detection thresholds were quite variable across participants between 10° and 80°, though they exhibit a similar shape, first increasing at smaller reference angles and then decreasing at larger reference angles. Evidently, acute angles are difficult to differentiate with nonconnected lines. Nevertheless, as in the connected lines condition, participants' precision increased around 90° compared with 80°/100° reference angles ($Z = 2.15$, $p = .016$; 8/8 participants showed the effect) and 55°/125° reference angles ($Z = 2.15$, $p = .016$; 8/8 participants showed the effect; see Table 2). We also observed significant asymmetries in detection thresholds, with better discrimination as deviants approached or crossed the 90° boundary in the 80°/100° reference blocks ($Z = 2.15$ $p = .016$, 8/8 participants showed the effect), but not in the 55°/125° reference blocks ($Z = 0.07$, $p = .473$, 4/8 participants showed the effect; see Figure 4B).

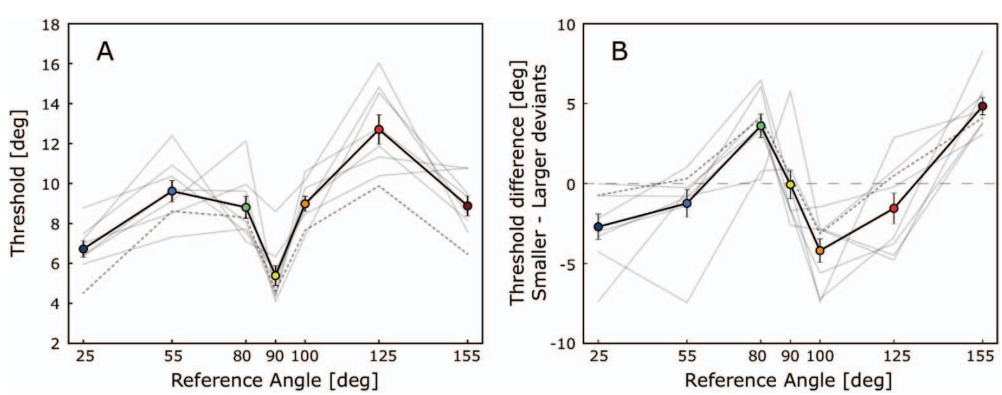

Figure 3. A. Estimated detection thresholds for each participant (gray lines) and on average (black line) at different reference angles for the connected lines condition. All curves are characterized by threshold drops toward 0°/180° as well as a sharp drop at 90°. The dashed curve corresponds to the participant shown in Figure 2. B. Asymmetries in the thresholds for detecting smaller versus larger deviants at different reference angles. Positive values on the *y*-axis indicate greater success in detecting larger versus smaller deviants, and negative values indicate the opposite. The dashed curve corresponds to the participant shown in Figure 2. For 80° and 100° references, acuity is better (smaller thresholds) when the deviant is in the direction of 90°. The asymmetry in detection thresholds crosses zero at almost exactly 90°, and there appears to be an approximately identical advantage on both sides of 90°. See the online article for the color version of this figure.

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Table 2
Group-Wise Mean (for the Connected Lines Condition) or Median (for the Nonconnected Lines) Detection Thresholds for Perpendicular and Parallel Reference Angles Compared With Other References and Detection Asymmetries

		Perpendicular	
	90	80/100	55/125
Connected lines—thresholds	5.4	8.9***	11.2***
Connected lines—asymmetry		toward: 6.9; away: 10.8**	toward: 11.1; away: 11.2
Nonconnected lines—thresholds	8.2	13.6*	20.7*
Nonconnected lines—asymmetry		toward: 10.1; away: 16.6*	toward: 19.5; away:19.0
		Parallel	
	0	10	25
Nonconnected lines—thresholds	2.5	13.3*	20.0*
Nonconnected lines—asymmetry		toward: 5.1; away: 21.2*	toward: 12.1; away: 28.4*

Note. Planned Holm-corrected, one-tailed *t*-tests (for the connected lines condition) or Wilcoxon signed-ranks tests (for the nonconnected lines condition) compare the 90° or 0° reference angles with the other reference angles and the magnitude of the asymmetry effects. All angle measures are in degrees.
* $p < .05$. ** $p < .01$. *** $p < .001$.

Discussion

This experiment presents two main findings. First, acuity in angle discrimination varies massively across angles. In particular, thresholds are smallest for parallels and perpendiculars. When angles are acute or obtuse and far from these two categories, thresholds of discrimination are high, though when angles deviate from such acute or obtuse angles and approach the parallel or perpendicular categories, discrimination becomes more precise. These effects persist over variations in the orientation and scale of the angle exemplars. While other studies reviewed above have found similar categorical responses to parallels and perpendiculars under some conditions, our results suggest that angle discrimination in general and the more precise discrimination of these angle categories in particular are invariant to scale and orientation.

How robust, then, are the categories of parallelism and perpendicularity in the face of more drastic variation in the angle stimuli, such as rotations in 3D depth, which are prevalent in everyday viewing conditions? In Experiment 1, where stimuli were presented in the frontoparallel plane, the higher precision for parallel and perpendicular lines could have arisen from a specific sensitivity at the retinal or retinotopic level. If, however, the effect arises at a more abstract representational level, that is, that parallelism and perpendicularity are properties that apply to the arrangement of lines in 3D space, then the effect should remain even

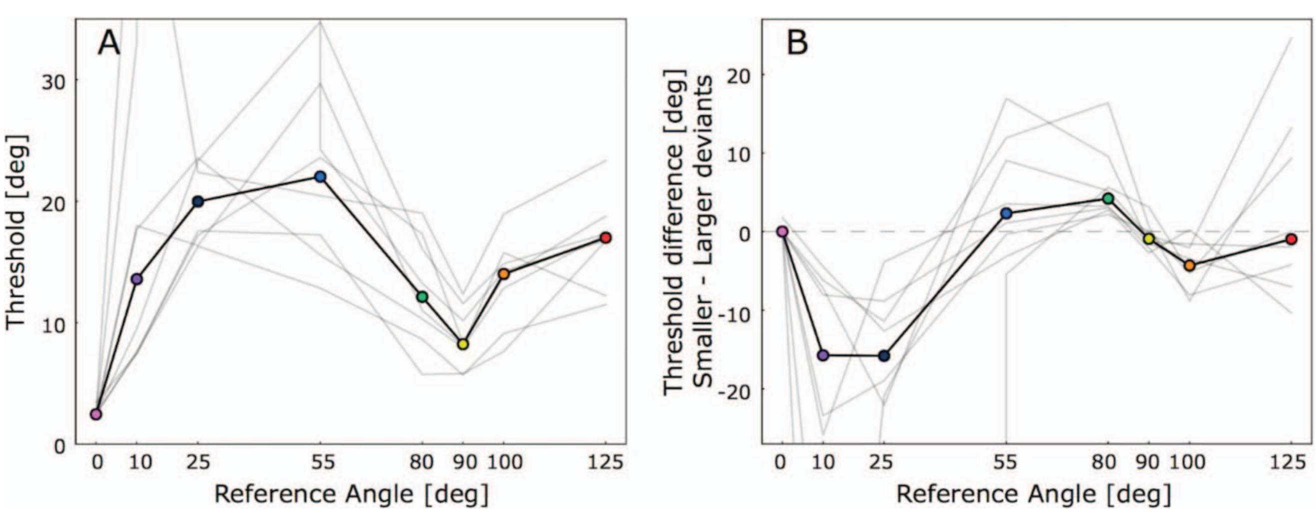

Figure 4. A. Estimated detection thresholds for each participant (gray lines) and the group medians (black line) at different reference angles for the nonconnected lines condition. Thresholds at some reference angles for some individuals exceed the graph limit. All curves are characterized by low thresholds for the 0° reference angle as well as a sharp drop at the 90° reference angle. B. Asymmetries in the detection thresholds for larger versus smaller deviants at different reference angles. The detection threshold asymmetry crosses zero at around 90°, with an approximately equal advantage on both sides. See the online article for the color version of this figure.

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if the stimuli are rotated in 3D depth, off the frontoparallel plane. This manipulation applies specifically to perpendicular lines: While parallel lines remain parallel even with rotation in depth (save a few, "accidental" viewpoints where their projections coincide: Amir, Biederman, & Hayworth, 2011, 2012; Biederman, Yue, & Davidoff, 2009), perpendicular lines vary greatly in their projected angle measure when the lines are rotated in depth (Nundy et al., 2000). Experiment 2 was therefore conducted both to evaluate how well the detection of perpendicular lines is preserved under viewing conditions that include additional rotations in 3D depth and also to examine whether perpendicularity as a category persists under these conditions.

Experiment 2

In this experiment, we presented new participants with stimuli derived from the connected lines condition of Experiment 1. In the *slanted screen* condition (Figure 5A), participants viewed the same stimuli as Experiment 1, but on a screen that was rotated in depth by 45°. In the *normal viewing* condition (Figure 5B), participants were presented with the same stimuli as Experiment 1 with an unrotated, frontoparallel screen (i.e., a direct replication of Experiment 1). By comparing participants' performance in these two conditions, we could evaluate whether both angle discrimination in general and the perpendicular category in particular persist with the rotation of our visual stimuli in depth. In the *projected stimuli* condition (Figure 5C), participants viewed the projection of the stimuli presented in the slanted screen condition on a frontoparallel screen. With this condition, we could evaluate whether the 3D context, presenting consistent cues to the reference frame transformation (like would be present in our everyday object recognition, see Nakayama, He, & Shimojo, 1995) or some general property of the stimuli's projection (which has a less clear connection to our everyday perceptual experience) might explain participants' responses in the slanted screen condition. If participants showed the same pattern of performance on the projected stimuli condition and the slanted screen condition, then there may be some geometric information at the level of the retinal projection that is guiding responses on the slanted screen condition.

Method

Participants. Twelve adults (10 women; M_{age} = 23 years; range 19–38 years) participated in this experiment. The sample size was set in advance of data collection. With the sample size and smallest effect size from Experiment 1 (N = 8, Cohen's d = 1.94), our power to detect group-wise categorical effects within condition would be .999. We decided to increase our sample size by 4 participants compared with Experiment 1 because we also planned to compare results across conditions in this experiment and investigate effects by condition. All participants had normal or corrected-to-normal vision and had completed high school. Most of them had also received a college or advanced degree. None were informed of the purpose and hypotheses of the study until after it was completed. Each provided informed consent prior to the experiment and were paid 10€ per hour plus an additional sum, which depended on their performance. On average, participants earned 72.95€ (range 70.47€–75.56€).

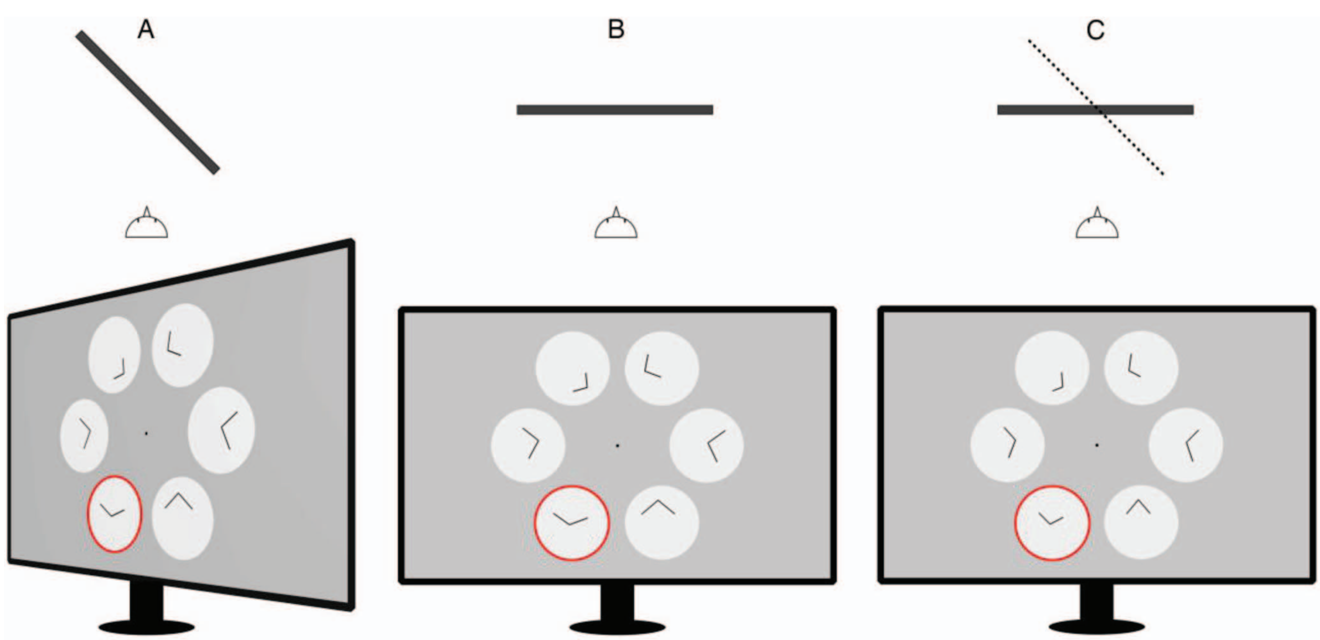

Figure 5. The top panel illustrates an overhead perspective of the setup for the screen and participant for the three conditions: A. *slanted screen*; B. *normal viewing*; C. *projected stimuli*. The lower panel illustrates the same trial (100° referent angle and 124° deviant) as it would be seen on the screen in each condition. The angles in the slanted screen condition are identical to the angles in the projected stimuli condition when projected on to the frontoparallel plane, as illustrated here. For illustration purposes, the correct response is circled in red. See the online article for the color version of this figure.

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Design, apparatus, procedure, and stimuli. The distance of the participant to the center of the screen, the lighting conditions, the placeholder sizes, the screen luminance, the presentation duration, and the stimuli parameters were identical in all conditions of Experiment 2 to the connected lines condition of Experiment 1, except that the lines that formed each angle were slightly shorter (uniform distribution from 1.6 to 3.5 dva). In the projected stimuli condition, the angles were transformed by reducing all horizontal coordinates of each angle by a factor of $cos\left(\frac{\pi}{4}\right)$, while keeping all vertical coordinates constant. This transformation was applied to the angles only so that the external reference frame in the slanted and projected stimuli conditions (placeholders' shapes and screen frame) provided cues to enforce the perception of a slanted or frontoparallel presentation of the stimuli. As in Experiment 1, trials were blocked by reference angle but fewer reference angles were presented to focus on the comparisons that were most relevant to exploring the categorical effect (see Table 1). Each type of deviant was presented 18 times in a random order in a block, three times at each of 6 possible target locations. The total number of trials per block thus ranged from 144 to 162.

The experiment was divided into three sessions, one for each of the three experimental conditions. The sessions took place on different days, 1 to 5 days apart, and the order of the condition presented at each session was counterbalanced across participants. Sessions started with a short training phase consisting of two trials per reference angle displaying ±24° angle deviants. Five blocks corresponding to the 5 reference angles followed the training phase. The order of the blocks within a session was chosen randomly for each participant. Each session lasted approximately 1.5 hr. As in Experiment 1, at the beginning of each block, participants were given a slide with instructions, showing an example of the reference angle in the vertical orientation. This slide was identical in the slanted screen, normal viewing, and projected stimuli conditions.

Analyses. As in Experiment 1, we fit performance with asymmetric down-pointing Gaussian curves. However, in several cases participants responded at chance (across participants: 10/120 blocks in the slanted screen condition; 3/120 blocks in the normal viewing condition; and 45/120 blocks in the projected condition; see Figure S3 for individual accuracy curves). Such at-chance performance resulted in very high and unreliable estimated threshold values. We thus again used nonparametric rank tests to minimize the impact of these extreme values in our comparisons, and we also conducted parametric analyses on accuracy data, which yielded identical results except where indicated (see online supplemental materials, Table S2).

As in Experiment 1, for all conditions of Experiment 2, we examined whether angle detection was more precise for the 90° reference angle compared with the other reference angles that were close to and far from 90° (i.e., 80°/100° reference angles and 55°/125° reference angles). We also evaluated whether deviant detection was more precise when deviants approached or crossed the 90° boundary compared with when they did not (all using Holm-corrected one-tailed Wilcoxon signed-ranks test). We only evaluated the asymmetries in the 80°/100° reference angles since the asymmetries in the 55°/125° reference angles were not significant in Experiment 1. By comparing the findings across the three conditions, first we asked whether categorical effects were present in the slanted screen condition, as in the normal viewing condition (we compared the thresholds and asymmetries in these two conditions using Scheirer-Ray-Hare [SRH] tests, a nonparametric equivalent of a two-way repeated measures ANOVA). Second, we asked whether the effects observed with the slanted screen were due to information present in the stimuli's projection rather than based on shape information invariant to 3D rotations (by comparing the slanted screen condition to the projected stimuli condition using a second SRH test).

Results

Figure 6A shows the median detection thresholds for each reference angle in all three conditions, and Table 3 shows tests for within-condition categorical effects. First, in the slanted screen condition, we observed more precise detection thresholds for 90°

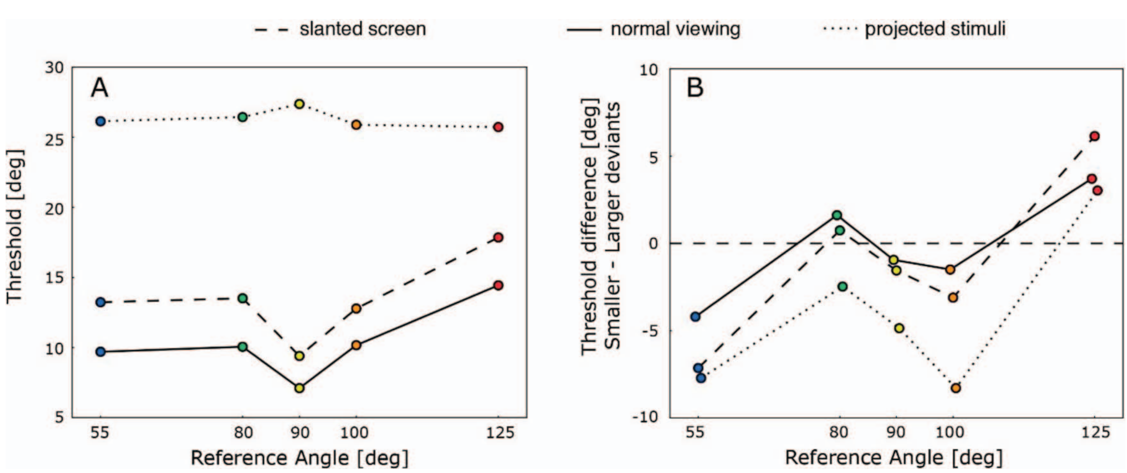

Figure 6. A. Median thresholds for different reference angles in the three conditions of Experiment 2. While the slanted screen and normal viewing conditions show a sharp drop at the 90° reference angle, the projected stimuli condition does not. B. Asymmetries in the threshold of detection for larger versus smaller deviants at different reference angles. See the online article for the color version of this figure.

Table 3
Group-Wise Median Detection Thresholds for Perpendicular Reference Angle Blocks Compared With Other Reference Blocks

	Perpendicular		
	90	80/100	55/125
Slanted screen—thresholds	9.4	13.2***	15.9***
Slanted screen—asymmetry		toward: 12.8; away: 14.8*	
Normal viewing—thresholds	7.1	10.6**	12.2**
Normal viewing—asymmetry		toward: 9.5; away: 11.3*	
Projected stimuli—thresholds	27.3	26.5	25.9
Projected stimuli—asymmetry		toward: 22.1; away: 28.1	

Note. Planned Holm-corrected, one-tailed Wilcoxon signed-ranks tests compare the 90° reference angles with the other reference angles. All angle measures are in degrees.
** $p < .01$. *** $p < .001$.

reference angles compared with both the 80°/100° reference angles ($Z = 3.18$, $p < .001$, 12/12 participants showed the effect), and also the 55°/125° reference angles ($Z = 3.18$, $p < .001$, 12/12 participants showed the effect). Second, this categorical effect did not differ across the slanted screen and normal viewing conditions. An SRH test comparing these two conditions revealed a significant main effect of reference angle ($H = 8.37$, $p = .004$), a significant main effect of condition ($H = 5.87$, $p = .015$), but, crucially, no interaction between reference angle and condition ($H = 0.20$, $p = .653$). As such, our normal viewing condition also replicated the results of Experiment 1, finding smaller detection thresholds with the 90° reference angle compared with the other reference angles (80°/100°: $Z = 2.47$, $p = .007$, 11/12 participants showed the effect; 55°/125°: $Z = 2.98$, $p = .001$, 11/12 participants showed the effect).

Does the increased performance with the 90° reference angle in the slanted screen condition result from an analysis of the stimuli as projected or from an invariance of the detection of perpendicularity over rotations in depth? If the former, then the same categorical effects should be present in the projected stimuli condition. An SRH test comparing the slanted screen and projected stimuli conditions revealed a significant main effect of reference angle ($H = 5.89$, $p = .015$), a significant main effect of condition ($H = 11.67$, $p < .001$), and an interaction between reference angle and condition ($H = 8.56$, $p = .003$). Indeed, contrary to the slanted screen condition, there was no sign of categorical effects in the projected stimuli condition; performance with the 90° reference angle was no different from the other reference angles (ps = 1.00).

Next, we assessed whether performance for reference angles 80°/100° was better when the deviant approached or crossed the 90° boundary (see Figure 6B). Performance was significantly asymmetric in the slanted screen condition ($Z = 1.85$, $p = .032$, 9/12 participants showed the effect). An SRH test comparing the thresholds toward and away from 90° in the slanted screen and normal viewing conditions revealed a significant main effect of deviant direction ($H = 5.00$, $p = .025$), condition ($H = 7.29$, $p = .007$), but no interaction between these two factors ($H = 0.02$, $p = .896$). In the normal viewing condition too, there was a significant asymmetry in detection thresholds ($Z = 1.85$, $p = .032$, 8/12 participants showed the effect). Thus, the asymmetry effects were comparable across the slanted screen and normal viewing conditions and were comparable to Experiment 1.

Finally, an SRH test comparing the slanted screen and projected stimuli conditions on the asymmetry of their deviant detection thresholds revealed a main effect of deviant direction ($H = 6.88$, $p = .009$), condition ($H = 11.38$, $p < .001$), and no interaction between the two factors ($H = 0.09$, $p = .770$). Indeed, asymmetries followed the same pattern in the projected stimuli condition as in the other conditions, with slightly better performance for angles deviating toward 90° versus away from 90°, though this difference was not significant ($Z = 1.20$, $p = .115$, 8/12 participants showed the effect; see online supplementary materials, Table S2). These weak asymmetry effects in the projected stimuli condition may have been due to a residual property of the angle transformation, rather than to participants' categorical discrimination of projected 90° angles. In particular, when the stimuli were transformed, the range of angle sizes presented on the screen differed based on the reference angle, with the greatest range occurring at 90°. Moreover, projected deviant angles fell outside the range of projected reference angles around 90° more often for deviants toward 90° than for deviants away from 90°, making deviants toward 90° easier to detect.

Discussion

This experiment builds on the findings of Experiment 1 by suggesting that both angle discrimination in general, and the perpendicular category in particular, are largely invariant to the rotation of shape stimuli in depth. A 45° rotation of the screen in the slanted screen condition degraded discrimination performance uniformly, yet left the relative discriminability of angles of different sizes intact, including preserving peak discriminability at 90°. Because the 90° angles in that condition were defined only in three dimensions with widely varying 2D projections, these results suggest that participants detected the angle intruder by monitoring the 3D angle sizes rather than their retinotopic, 2D projections. This conclusion is further supported by comparisons between the slanted screen condition and the projected stimuli condition. Here, the 2D angles that were projected on the retina were identical between conditions, yet accuracy varied greatly, suggesting that the preserved shape discrimination in the slanted screen condition was not due to properties of the 2D projection of the stimuli. Indeed, participants performed so poorly in the projected stimuli condition

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that the data were difficult to model, and it is thus hard to make specific conclusions about the characteristics of our shape discrimination with stimuli transformed in this way. By contrast, rotating shapes in depth, with all of the real-world depth cues intact, led to a largely preserved ability to discriminate angles.

General Discussion

Across two experiments presenting angle-intruder detection tasks, we demonstrated that the discriminability of both connected and nonconnected lines forming different angles varies greatly depending on the size of the angles being discriminated. Most notably, thresholds of discrimination were significantly smaller when lines formed parallels and perpendiculars. When angles were acute or obtuse and far from these two special angle categories, thresholds of discrimination were high, though when angles deviated from nearer acute or obtuse angles to approach the parallel or perpendicular categories, discrimination became more precise. This pattern of results persisted not only over variations in the orientations and scale of the angle exemplars, but also over their rotations in depth. In particular, when the very same angle stimuli were presented on a screen rotated 45° in depth, the relative discriminability of angles of different sizes persisted. Most remarkably, the significantly smaller threshold for detecting perpendicular angles persisted, despite the variability in the 2D projections of these angles, and this result was not due to properties of the stimuli's 2D projection. Our findings thus suggest that angle discrimination in general and the special angle categories of parallelism and perpendicularity in particular affect participants' shape judgments through their real-world angle information.

The prior literature had outlined different models of angle discrimination to which we can compare our results. In two cases, more precise discrimination for horizontal and vertical lines was invoked to explain better acuity for 90° and 0°/180° angles: Chen and Levi (1996) suggested an orientation-independent, Weber-like discrimination space except at 90°, where discrimination is heightened and rooted in the presence of vertical and horizontal lines; Xu, Chen, and Kuai (2018) proposed that angles which include either a vertical or horizonal line are discriminated most precisely. In contrast, Heeley and Buchanan-Smith (1996) proposed that, unlike the discrimination of individual line orientations, angle discrimination operates over a reference frame that is object-centered, like the discrimination of other, more complex objects. Our findings thus support and extend this last model: Not only did our stimuli specifically avoid using lines at or near the horizontal or vertical, but also more precise discrimination persisted when the angles were rotated in depth, suggesting that angle discrimination unfolds at the level of the 3D angle size, as it might for the shape of complex 3D objects.

What then is the status of the geometric categories of *parallelism* and *perpendicularity*? Some research has suggested that sensitivity to parallelism, at least, arises early in child development. Even 4-year-old children indicate that a pair of parallel lines is the "most different" from 5 other pairs of lines that present continuous differences in angle at varied absolute orientations and scales (Izard, Pica, Dehaene, Hinchey, & Spelke, 2011a, 2011b). Children's knowledge of the word "parallel," moreover, has no relation to this choice (Izard et al., 2011a, 2011b). As such, children's judgments of parallelism may be rooted in the recognition of more basic shape properties, for example, that two lines have the same orientation or that they maintain a constant distance from each other—and perhaps these very properties contribute to the categorical effects for parallels documented here. Future research should explore whether children display the same categorical effects around the discrimination of parallels as adults and whether their specialized treatment of parallel lines changes with more visual experience or with explicit learning of formal geometry in school.

What about perpendicularity, whose angle measure changes greatly with rotations in 3D depth? Unlike their performance with parallel lines, not until age 7 or older do children pick out a pair of perpendicular lines from other pairs of lines that vary continuously in angle, and individual children's knowledge of the lexical terms "right angle" and "perpendicular" correlates with this choice (Izard et al., 2011a). The present task with adults did not rule out the possibility that explicitly learned conceptual representations were supporting participants' performance. All the adults in this study had benefitted from a formal education at least through high-school, which likely included a geometry class where the concepts of parallelism and perpendicularity were taught. In particular, since participants were introduced to each block of trials with an image that depicted the reference angle, it was possible that, for the parallel and perpendicular categories, participants labeled the reference and used that label to access a stored representation of an exemplar from that category from memory, making its recognition more accurate (Firestone & Scholl, 2016). Like other studies examining advantages in discrimination for complex spatial stimuli, defined, for example, by topological relations, it is unknown the extent to which the present findings reflect verbal coding or pure visual processing (Lovett & Franconeri, 2017). Indeed, a suite of studies using search tasks and deviant detection tasks show enhanced detection of visual stimuli including colors, simple shapes, and more complex objects when category boundaries line up with linguistic labels (see Goldstone & Hendrickson, 2010; Lupyan, 2012). Future studies using the same angle stimuli with adults from other cultures, children, and nonhuman animals might shed further light on whether the effects observed in this study were perceptual or cognitive.

Nevertheless, the detection of perpendicular angles may indeed be universal. A group of adults from the Munducuru tribe in the Amazon, who, unlike the adults in the present sample, have no specialized geometric training or vocabulary, pick out a pair of perpendicular lines as the "most different" from other pairs of lines that vary continuously in angle (Izard et al., 2011a). While categorization of perpendiculars (as in this study with the Munducuru adults, Izard et al., 2011a) and categorical effects in discrimination (as measured with the adults in this article) may dissociate, it is possible that a perceptually based perpendicular category arises spontaneously in development and that education in formal geometry refines it or heightens attention to it (see Piazza, Pica, Izard, Spelke, & Dehaene, 2013 for further exploration of this suggestion in the numerical domain). If this is the case, it remains a challenge to identify a perceptual learning mechanism that might operate over such a long period of development and create such a robust category.

Might visual experience through development bolster our recognition of 3D shape information from which a heightened sensitivity to perpendicular angles could emerge? Previous work has

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shown that high-level perceptual learning can occur over extended periods of time, for example, to create or refine specific object categories like letters and faces (Dehaene, Cohen, Sigman, & Vinckier, 2005; Maurer, Grand, & Mondloch, 2002). This kind of perceptual learning is nevertheless limited, in that categorical effects disappear when, for example, face stimuli are inverted (Maurer et al., 2002). Such limits may be adaptive to the context: While face recognition is not invariant to large 2D orientation changes, it is invariant over large changes in viewpoint (i.e., in full view or in profile), expression, and lighting (Anselmi & Poggio, 2014).

The perceptual learning associated with 3D object recognition may accumulate 2D and 3D rotational invariance through development since objects are often seen rotated in 2D or 3D space. While newborns differentiate among 2D angle shapes given long periods of habituation (Slater, Mattock, Brown, & Bremner, 1991), 7-month-old infants fail to differentiate shapes differing in angle during brief exposures to 2D forms with simultaneous variations in scale, direction, and orientation (Dillon, Izard, & Spelke, 2019). Research with toddlers has suggested that 3D object recognition undergoes protracted development, with a spurt in the ability to recognize objects by their 3D geometric shapes between the ages of 18–24 months (Augustine, Smith, & Jones, 2011; Smith, 2009), and studies with older children have shown that the period during which we become better able to recognize unfamiliar viewpoints of 3D objects captured in 2D drawings extends to adulthood (Jüttner, Müller, & Rentschler, 2006; Jüttner, Wakui, Petters, Kaur, & Davidoff, 2013; Landau, Hoffman, & Kurz, 2006). Further developmental work, investigating invariances in angle detection may begin to shed light on how such shape representations become more robust to everyday 3D viewing conditions.

Even if angle detection in general becomes more robust to rotations in depth through development, such development alone does not explain the emergence of a perpendicular category. A rotationally invariant representation of perpendicularity could be singled out because a perpendicular line is the most symmetrical position for a line relative to another line (i.e., splitting a line in half) or to a plane, and both infants and young children show some sensitivity to symmetry (Bornstein, Ferdinandsen, & Gross, 1981; Huang, Xue, Spelke, Huang, Zheng, & Peng, 2018). A rotationally invariant representation of perpendicularity could also derive from representations of the absolute vertical and horizontal. These orientations are robustly represented early in development and across animal species (Appelle, 1972). For example, 5-month-old infants take longer to look toward a deviant oblique line among other oblique lines versus among vertically oriented lines the same angle measure away (Franklin, Catherwood, Alvarez, & Axelsson, 2010).

Finally, the present study raises questions about the relations between our perception of angles and our conception of formal Euclidean geometry. Euclid famously did not explain what he means by "equal" angles when, at the beginning of his *Elements* (Book 1, Definition 10), he defines a *perpendicular* as a line with equal angles to either side as it stands up from a baseline. This definition is strikingly perceptual: We see what he means. Do we need to see the abstractions of geometry to conceive of them? Are they driven by mental imagery of specific spatial exemplars, abstract idealized concepts, or linguistic representations? Indeed, what are the mental representations guiding our geometric reasoning? Individuals differ in their ability to discriminate angles: Might individual differences in our angle discrimination predict our ability to reason about the properties of points, lines, and figures on the Euclidean plane? Could training in angle discrimination cause short- or long-term benefits to geometric reasoning or judgment? We are only just beginning to probe how the human mind navigates the perceptual and conceptual worlds of geometry.

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